Advanced Robotics—Numerical Solution for the IKP of Serial Manipulators

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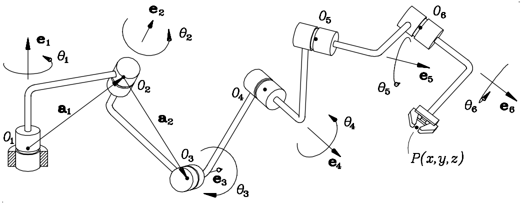
Human and Robot Interaction Laboratory



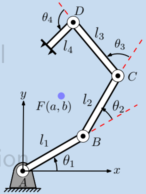
Human and Robot Interaction Laboratory

April 8, 2013

The Numerical Solutions for the IKP

Introduction

- We pause here what we have seen for the redundant serial manipulator.



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- For the decoupled a explicit solution can be presented
- It is of great help for non-decoupled serial manipulators.

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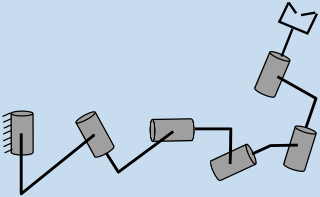

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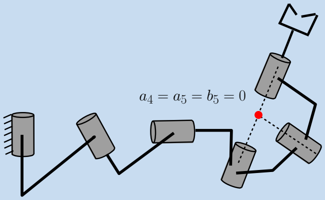

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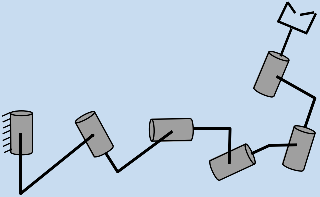


The Numerical Solutions for the IKP

The Procedure

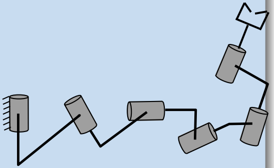
- We have to define the D-H parameters and the following relations:

$$[\mathbf{Q}_i]_i = \begin{bmatrix} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i \\ \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i \cos\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i \end{bmatrix}$$

$$[\mathbf{a}_i]_i = \begin{bmatrix} a_i \cos\theta_i \\ a_i \sin\theta_i \\ b_i \end{bmatrix}$$

$$\mathbf{Q} = \mathbf{Q}_1\mathbf{Q}_2\mathbf{Q}_3\mathbf{Q}_4\mathbf{Q}_5\mathbf{Q}_6$$

$$\mathbf{p} = \sum_{1}^{6} [\mathbf{a}_i]_1$$



The Numerical Solutions for the IKP

The Procedure

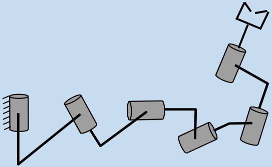
- We define $\mathbf{f}_A$ and $\mathbf{f}_B$ as follows:

$$\mathbf{f}_A = \mathbf{Q}_1\mathbf{Q}_2\mathbf{Q}_3\mathbf{Q}_4\mathbf{Q}_5\mathbf{Q}_6 - \mathbf{Q}$$

$$\mathbf{f}_B = \sum_1^6 [\mathbf{a}_i]_1 - \mathbf{p}$$

- Then $\mathbf{f}$ is defined as follows:

$$\mathbf{f} = \begin{bmatrix} \mathbf{f}_A \\ \mathbf{f}_B \end{bmatrix}$$



The Numerical Solutions for the IKP

The Procedure

- We define the following:

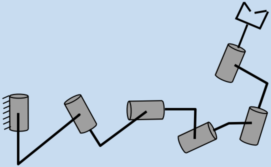
$$\mathbf{x}_{k+1} = \mathbf{x}_k + \Delta\mathbf{x}_k$$

- Then upon using the Taylor series:

$$\mathbf{f}(\mathbf{x}_{k+1}) = \mathbf{f}(\mathbf{x}_k + \Delta\mathbf{x}_k)$$
$$= \mathbf{f}(\mathbf{x}_k) + \mathbf{J}(\mathbf{x}_k)\Delta\mathbf{x}_k = 0$$

- which leads to

$$\mathbf{J}(\mathbf{x}_k)\Delta\mathbf{x}_k = -\mathbf{f}(\mathbf{x}_k)$$



$$\Delta\mathbf{x}_k = -\mathbf{J}^\dagger(\mathbf{x}_k)\mathbf{f}(\mathbf{x}_k)$$
$$\mathbf{J}^\dagger = (\mathbf{J}^T\mathbf{J})^{-1}\mathbf{J}^T$$
$$\mathbf{f}^T\mathbf{f} < \epsilon$$

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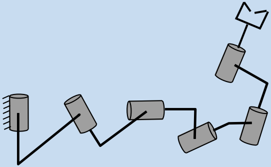

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The Numerical Solutions for the IKP

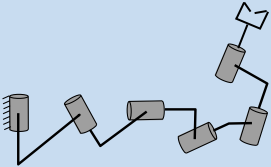
The Procedure

- There is another approach:

$$\min_{x} z \quad \text{where} \quad z = \frac{1}{2}\mathbf{f}^T\mathbf{f}$$

where

$$\nabla z = \frac{\partial z}{\partial \mathbf{x}} = \left(\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right)^T \frac{\partial z}{\partial \mathbf{f}} = \mathbf{J}^T\mathbf{f} = \mathbf{0}$$



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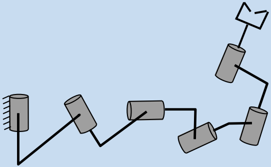

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The Numerical Solutions for the IKP

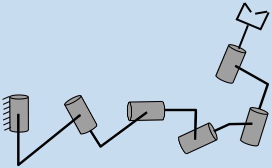
An alternative to express f

- From the linear invariant, we define the following:

$$\mathbf{f}_R(\boldsymbol{\theta}) = 2\text{vect}(\mathbf{Q}_1 \ldots \mathbf{Q}_6) - 2\text{vect}(\mathbf{Q}) = 0$$

$$f_S(\boldsymbol{\theta}) = \text{tr}(\mathbf{Q}_1 \ldots \mathbf{Q}_6) - \text{tr}(\mathbf{Q}) = 0$$

$$\mathbf{f}_T(\boldsymbol{\theta}) = \sum_1^6 [\mathbf{a}_i]_1 - \mathbf{p} = 0$$



$$\mathbf{J} = \frac{\partial \mathbf{f}}{\partial \boldsymbol{\theta}} = \begin{bmatrix} \frac{\partial \mathbf{f}_R(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \\ \frac{\partial \mathbf{f}_S(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \\ \frac{\partial \mathbf{f}_T(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \end{bmatrix}$$

- Then we define the following:

$$\mathbf{f}(\boldsymbol{\theta}) = [\mathbf{f}^T(\boldsymbol{\theta}), f_S(\boldsymbol{\theta}), \mathbf{f}_T^T(\boldsymbol{\theta})]^T$$

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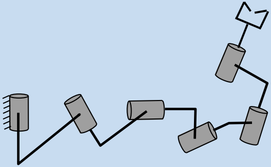

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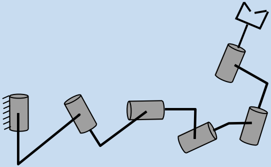
An alternative to express f

- It can be shown that:

$$\frac{\partial f_R(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} = (\mathbf{1}\mathrm{tr}(\mathbf{Q}) - \mathbf{Q})\mathbf{A}$$

$$\frac{\partial f_S(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} = -2\mathbf{q}^T\mathbf{A}$$

$$\frac{\partial f_T(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} = \mathbf{B}$$



- where

$$\mathbf{A} \equiv [\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_n]$$

$$\mathbf{B} \equiv [\mathbf{e}_1 \times \mathbf{r}_1, \mathbf{e}_2 \times \mathbf{r}_2, \ldots, \mathbf{e}_n \times \mathbf{r}_n]$$

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